II Semester M.Sc. Degree Examination, June/July 2014
(NS) (2006 Scheme)
MATHEMATICS
M-202: Complex Analysis

Time : 3 Hours Max. Marks : 80

Instructions : *i) Answer **any five** full questions choosing at least **two** from **each** Part.*
*ii) **All** questions carry **equal** marks.*

PART – A

1. a) Find the radius of convergence of

i) $\sum_{n=0}^{\infty} \left[3+(-1)^n\right]^{1/n}$

ii) $1+\frac{a.b}{1.c}z+\frac{a(a+1)\,b(b+1)}{1.2c(c+1)}z^2+...$

b) If R is the radius of convergence of $\sum a_n z^n$, then prove the following :

i) The power series converges for $|z| < R$ and diverges for $|z| \geq R$.

ii) If $0 < \rho < R$ the power series converges uniformly in $\{|z| \leq \rho < R\}$.

c) Prove that the power series and its derivative have the same radius of convergence. **(4+4+8)**

2. a) State and prove Taylor's theorem for an analytic function f (z) in a region D about the point z = 0 on D.

b) Find the Laurent's expansion of $f(z)=\sinh\left(z+\frac{1}{z}\right)$ for $|z| > 0$.

c) State and prove the Cauchy's integral formula and use it to evaluate

$$\int_{|z|=1} \frac{\cos 2\pi z}{(2z-1)(z-3)}dz.$$

(4+4+8)

3. a) State and prove Cauchy's theorem for triangle.

   b) Let f (z) be analytic function having an isolated singularity at z = a. If |f(z)| is bounded in a neighbourhood {0<|z – a| < r} Then prove that f (z) has a removable singularity at z = 0.

   c) Define the terms :

      i) Pole

      ii) Removable singularity

      iii) Essential singularity

      iv) Isolated singularity and give examples for each. **(8+4+4)**

4. State and prove Hadmard's three circle theorem and prove that log M (r) is a convex function of log r. **16**

PART – B

5. a) Define holomorphic and meromorphic functions with examples.

   b) State and prove the Cauchy's residue theorem.

   c) Find the residue at the poles of the function f (z) given by

$$f(z) = \frac{\sin z}{z^2(z-1)^2(z-2)^3}.$$ **(4+6+6)**

6. a) Evaluate :

   i) $\int_0^{2\pi} \frac{\sin^2\theta}{a+b\cos\theta}\,d\theta, \quad a > b > 0$ ii) $\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2+1)^2}\,dx$

   b) State and prove the argument principle theorem. **(5+5+6)**

7. a) State and prove the Rouche's theorem.

   b) Using the result of the Weistrass factorization theorem, construct an entire function having zero's at 1, 2, 3.

   c) State and prove Phragmen-Lindelof theorem. **(6+3+7)**

8. a) Define Harmonic function. State and prove the mean value property for harmonic functions.

   b) Derive the Poisson's integral formula on standard notations. **(6+10)**
